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FORESTS WHOSE INDEPENDENCE POLYNOMIALS ARE PALINDROMIC AND HAVE ONLY REAL ROOTS*

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ABSTRACT. A *stable set* (or an *independent* set) in a graph G is a set of pairwise non-adjacent vertices, and the *independence number* $\alpha(G)$ is the cardinality of a maximum stable set in G. The *independence polynomial* of G is

$$I(G;x) = s_0 + s_1x + s_2x^2 + \cdots + s_\alpha x^\alpha, \alpha = \alpha(G),$$

where s_k is the number of stable sets of size k in G (I. Gutman and F. Harary, 1983). If $s_i = s_{\alpha-i}, 0 \leq i \leq \lfloor \alpha/2 \rfloor$, then $I(G;x)$ is called *palindromic*. Unlike the matching polynomial, the independence polynomial of a graph can have non-real roots.

In this paper, we show that for every integer $2 \leq \alpha \neq 3$ there is a forest F consisting of at most two non-trivial trees, whose $\alpha(F) = \alpha$, and $I(F;x)$ is palindromic and has only real roots.

1. INTRODUCTION

Throughout this paper $G = (V, E)$ is a finite, undirected, loopless and without multiple edges graph, whose vertex set is $V = V(G)$ and edge set is $E = E(G)$. $G[X]$ is the subgraph of G induced by $X \subset V$, while $G - X$ means the subgraph $G[V - X]$. We also denote by $G - F$ the partial subgraph of G obtained by deleting the edges of F, for $F \subset E(G)$, and we write shortly $G - e$, whenever $F = \{e\}$. The *neighborhood* of a vertex $v \in V$ is the set $N_G(v) = \{w : w \in V \ \ \textit{and } vw \in E\}$, and $N_G[v] = N_G(v) \cup \{v\}$; if there is no ambiguity on G, we use $N(v)$ and $N[v]$, respectively. A vertex v is *pendant* if its neighborhood contains only one vertex; an edge $e = uv$ is *pendant* if one of its endpoints is a pendant vertex. K_n, P_n, C_n denote respectively, the complete graph on $n \geq 1$ vertices, the chordless path on $n \geq 1$ vertices, and the chordless cycle on $n \geq 3$ vertices. A forest (tree) is called *trivial* if it has no edges. The *disjoint union* of G_1, G_2 is the graph $G_1 \cup G_2$ having as a vertex set the disjoint union of $V(G_1), V(G_2)$, and as an edge set the disjoint union of $E(G_1), E(G_2)$. In particular, nG denotes the disjoint union of $n \geq 1$

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copies of G. The *Zykov sum* of two disjoint graphs G_1, G_2 is the graph $G_1 + G_2$ with $V(G_1) \cup V(G_2)$ as a vertex set and $E(G_1) \cup E(G_2) \cup \{v_1 v_2 : v_1 \in V(G_1), v_2 \in V(G_2)\}$ as an edge set.

By a *stable* (*or an independent*) set in G we mean a set of pairwise non-adjacent vertices. A stable set of maximum size will be referred to as a *maximum stable set* of G, and $\alpha(G)$ is the size of a maximum stable set in G. A graph having only one maximum stable set is called a *unique independence graph*.

If s_k equals the number of stable sets of size k in a graph G, then the polynomial

$$I(G; x) = s_0 + s_1 x + s_2 x^2 + \cdots + s_\alpha x^\alpha, \alpha = \alpha(G),$$

is called the *independence polynomial* of G, (Gutman and Harary, [14]), or the *independent set polynomial* of G (Hoede and Li, [18]). The reader is referred to [26] for a survey on this graph polynomial.

Proposition 1.1. [14] *The following equalities are true:*
(i) $I(G_1 \cup G_2; x) = I(G_1; x) \cdot I(G_2; x)$;
(ii) $I(G_1 + G_2; x) = I(G_1; x) + I(G_2; x) - 1$;
(iii) $I(G; x) = I(G - v; x) + x \bullet I(G - N[v]; x)$ *holds for each* $v \in V(G)$.

A polynomial $a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$, whose coefficients are real, is called:
- *palindromic* if $a_i = a_{n-i}, i = 0, 1, \ldots, \lfloor n/2 \rfloor$;
- *unimodal* if there is some $k \in \{0, 1, \ldots, n\}$ such that

$$a_0 \leq \cdots \leq a_{k-1} \leq a_k \geq a_{k+1} \geq \cdots \geq a_n;$$

- *log-concave* if $a_i^2 \geq a_{i-1} \cdot a_{i+1}, i \in \{1, 2, \ldots, n-1\}$.

For instance, the independence polynomial
- $I(K_{42} + 3K_7; x) = 1 + 63x + 147x^2 + 343x^3$ is log-concave and has only one real root;
- $I(K_{43} + 3K_7; x) = 1 + 64x + 147x^2 + 343x^3$ is unimodal, but non-log-concave, because $147^2 - 64 \cdot 343 = -343 < 0$;
- $I(K_{127} + 3K_7; x) = 1 + 148x + 147x^2 + 343x^3$ is non-unimodal.

It is worth mentioning that non-isomorphic graphs, connected or not, can have the same independence polynomial, even palindromic and having only real roots.

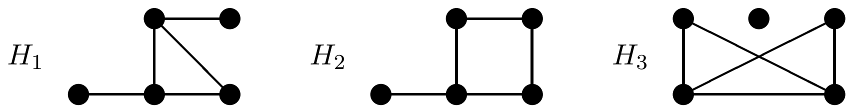

FIGURE 1. Non-isomorphic graphs having the same independence polynomial

For instance, the graphs from Figure 1 are clearly non-isomorphic, while

$$I(H_1; x) = I(H_2; x) = I(H_3; x) = 1 + 5x + 5x^2 + x^3 = (1 + x)(1 + 4x + x^2).$$

The unimodality of independence polynomials was analyzed in a number of papers, like [1], [3], [11], [16], [21], [22], [23], [24], [25], [27], [31].

It is known that each log-concave polynomial of positive coefficients is also unimodal. A result, due to I. Newton, ensures that each polynomial, that has non-negative coefficients and only real roots, is log-concave, and hence unimodal, as well.

Independence polynomial was defined as a generalization of matching polynomial of a graph, because the matching polynomial of a graph G and the independence polynomial of its line graph are identical.

Recall that given a graph G, its *line graph* $L(G)$ is the graph whose vertex set is the edge set of G, and two vertices of $L(G)$ are adjacent if they share, as edges, an end in G.

FIGURE 2. G_1 and its line graph $L(G_1) = G_2$

For instance, the graphs G_1 and G_2 depicted in Figure 2 satisfy $G_2 = L(G_1)$ and, hence,

$$I(G_2; x) = 1 + 7x + 12x^2 + 4x^3 = M(G_1; x),$$

where $M(G_1; x)$ is the matching polynomial of the graph G_1. Heilmann and Lieb proved the following assertion.

Theorem 1.1 ([17]). *The roots of the matching polynomial of a graph are real.*

Taking into account the connection between the matching polynomial of a graph G and the independence polynomial of its line graph $L(G)$, one can assert that $I(L(G); x)$ has only real roots, for every graph G. Nevertheless, the independence polynomial of a graph can have non-real roots and this is true also for trees, e.g., $I(K_{1,3}; x) = 1 + 4x + 3x^2 + x^3$.

Clearly, each real root of $I(G; x)$ is negative, since all the coefficients of $I(G; x)$ are positive. Moreover, it is known that a root of smallest modulus of $I(G; x)$ is real, for any graph G, [5]. In fact, $I(G; x)$ has only one root of smallest modulus [10].

A graph is called *claw-free* if it has no induced subgraph isomorphic to $K_{1,3}$. Hamidoune [16] proved that the independence polynomial of a claw-free graph is log-concave, and he conjectured that for every claw-free graph, the roots of its independence polynomial are all real. Chudnovsky and Seymour proved this assertion, thus extending Theorem 1.1, since line graphs are claw-free.

Theorem 1.2 ([8]). *The independence polynomial of a claw-free graph has only real roots.*

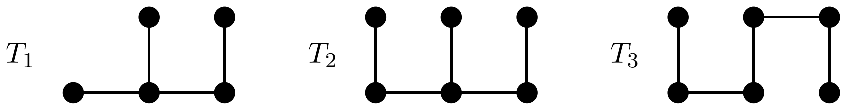

FIGURE 3. Each T_i has $|\Omega\,(T_i)| \geq 2$ and $\alpha(T_i) = 3, 1 \leq i \leq 3$

The independence polynomials of the trees from Figure 3 are respectively,

$$I(T_1; x) = 1 + 5x + 6x^2 + 2x^3,$$
$$I(T_2; x) = 1 + 6x + 10x^2 + 5x^3,$$
$$I(T_3; x) = 1 + 6x + 10x^2 + 4x^3$$

and all have only real roots.

Brown *et al.* [5] showed that the independence polynomial of any graph G with $\alpha(G) = 2$ has real roots. The assertion fails for graphs having stability number greater than 2, even for trees. For instance, the trees from Figure 4 are unique independence graphs, their independence polynomials are

$$I(P_5; x) = 1 + 5x + 6x^2 + x^3 \text{ and } I(K_{1,3}; x) = 1 + 4x + 3x^2 + x^3,$$

but only $I(P_5; x)$ has all the roots real.

FIGURE 4. $\alpha(P_5) = \alpha(K_{1,3}) = 3$ and P_5 and $K_{1,3}$ are unique independence graphs

Alavi *et al.* proved that for any permutation π of $\{1, 2, \ldots, \alpha\}$ there is a graph G with $\alpha(G) = \alpha$ such that $s_{\pi(1)} < s_{\pi(2)} < \cdots < s_{\pi(\alpha)}$ (see [1]). Nevertheless, for trees, they stated the following conjecture, which is still open.

CONJECTURE 1.1 ([1]). *The independence polynomial of every tree is unimodal.*

It is worth noticing that if $\alpha(G) \leq 3$ and $I(G; x)$ is palindromic, then it is also log-concave. However, there exist graphs with stability number ≥ 4, whose independence polynomials are palindromic and even non-unimodal. For instance,

$$I(K_{52} + 3K_4 + 4K_1; x) = 1 + 68x + 54x^2 + 68x^3 + x^4,$$
$$I(H; x) = 1 + 2406x + 1382x^2 + 1382x^3 + 2406x^4 + x^5,$$
$$\text{where} H = K_{1832} + 4K_7 + (K_2 \cup K_{539}) + 5K_1$$

Taking into account that $s_0 = 1$ and $s_1 = |V(G)| = n$, it follows that the palindromicity of $I(G; x)$ implies that $s_0 = s_{\alpha(G)}$ and $s_1 = s_{\alpha(G)-1}$, i.e., G has only one maximum stable set, say S, and $|V(G)| - \alpha(G)$ stable sets, of size $\alpha(G) - 1$, that are not subsets of S. Hence, G must be a unique independence graph.

Theorem 1.3. (i) [29] *The product of two palindromic polynomials is palindromic.*
(ii) [2] *If P and Q are both unimodal and palindromic, then $P \cdot Q$ is unimodal.*

The product of two polynomials, one log-concave and the other unimodal, is not always log-concave, for instance, if $G = K_{40} + 3K_7, H = K_{110} + 3K_7$, then

$$\begin{aligned} I(G;x) \cdot I(H;x) &= \left(1 + 61x + 147x^2 + 343x^3\right)\left(1 + 131x + 147x^2 + 343x^3\right) \\ &= 1 + 192x + 8285x^2 + 28910x^3 + 87465x^4 + 100842x^5 + 117649x^6, \end{aligned}$$

which is not log-concave, because $100842^2 - 87465 \cdot 117649 = -121\,060\,821$. However, the following result, due to Keilson and Gerber, gives a sufficient condition for two polynomials to have a unimodal product.

Theorem 1.4 ([19])**.** *If P is log-concave and Q is unimodal, then $P \cdot Q$ is unimodal, while the product of two log-concave polynomials is log-concave.*

The *corona* of the graphs G and H is the graph $G \circ H$ obtained from G and $|V(G)|$ copies of H, such that each vertex of G is joined to all vertices of a copy of H. The connection between the independence polynomials of G, H and $G \circ H$ is given by the following result, due to I. Gutman.

Theorem 1.5 ([13])**.** *If G is a graph of order n, then*

$$I(G \circ H; x) = (I(H;x))^n \cdot I\left(G; \frac{x}{I(H;x)}\right).$$

For example, if $G = \overline{K_n}$, then $\alpha(G) = |V(G)| = n$ and $G \circ K_p = nK_{p+1}$ has $I(G \circ K_p; x) = (1 + (p+1) \cdot x)^n$, whose roots are all real.

The palindromicity of matching polynomial of a graph was investigated in [20], while for independence polynomial we cite [11], [12] and [30]. In [4] are build, by means of a recursive "path-like" construction, a family of graphs whose independence polynomials are unimodal and palindromic. Three ways to build graphs having palindromic independence polynomials are presented in [30]. Recall the following result.

Theorem 1.6 ([30])**.** *The polynomial $I(G \circ 2K_1; x)$ is palindromic for every graph G.*

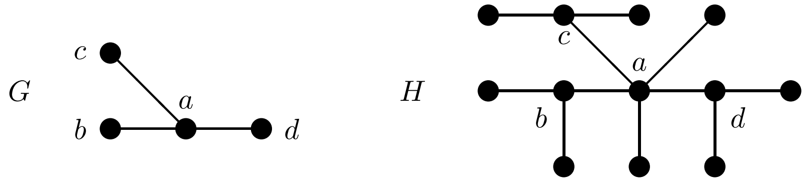

FIGURE 5. The graphs G and $H = G \circ 2K_1$

For instance, the graphs from Figure 5 have: $I(G;x) = 1+4x+3x^2+x^3$, while, by Theorem 1.5, we get

$$\begin{aligned} I(H;x) = I(G \circ 2K_1;x) &= [I(2K_1;x)]^4 \cdot I\left(G; \frac{x}{I(2K_1;x)}\right) \\ &= (1+x)^2 \bullet \left(1+10x+34x^2+51x^3+34x^4+10x^5+x^6\right) \\ &= 1+12x+55x^2+129x^3+170x^4+129x^5+55x^6+12x^7+x^8, \end{aligned}$$

which is clearly palindromic. Notice that none of the polynomials $I(G;x)$ and $I(H;x)$ has all the roots real.

In this paper we show that for each positive integer $\alpha \neq 3$, there is a forest F consisting of at most two trees, each different from K_1, such that: $\alpha(F) = \alpha$ and its independence polynomial $I(F;x)$ is palindromic and has only real roots.

2. RESULTS

As for other graph polynomials, such as matching polynomial, chromatic polynomial, it is natural to ask about the nature and location of the roots of independence polynomial. This problem was investigated in a number of papers, like [5], [7], [6], [8], [9], [10], [15].

It is easy to see that $\alpha(G \circ K_p) = |V(G)|$ holds for every graph G. As an example, the graph $P_n \circ K_1$ has $\alpha(P_n \circ K_1) = |V(P_n)| = n$. Moreover, its independence polynomial $I(P_n \circ K_1;x)$ is log-concave [24].

Let us observe that $\alpha(\overline{K_n}) = \left|V(\overline{K_n})\right| = n$ and $I(\overline{K_n};x) = (1+x)^n$. In addition, one can say that $\overline{K_n} \circ 2K_1 = n(K_1 \circ 2K_1) = nP_3$ and, therefore,

$$I(\overline{K_n} \circ 2K_1;x) = (I(P_3;x))^n = (1+3x+x^2)^n$$

which, clearly, has all its roots real and, by Theorem 1.3, is palindromic, as well.

Theorem 2.1 ([28])**.** *Let G be a graph with non-empty edge set. If all the roots of $I(G;x)$ are real, then*

(i) *$I(G \circ K_p;x)$ has all the roots real;*

(ii) *$I(G \circ 2K_1;x)$ is palindromic and has only real roots.*

The case of $I(G \circ K_p;x)$ with $p = 1$ was treated in [25].

Notice that $G = nK_1$ is a trivial forest (since it has no edge), whose independence polynomial $I(G;x) = (1+x)^n$ is palindromic and all its roots are real.

Proposition 2.1. *There exists a forest F, having $\alpha(F) = 3$, whose independence polynomial is palindromic and has only real roots.*

Proof. The forest $F = K_1 \cup P_3$ has $\alpha(F) = 3$ and its independence polynomial

$$I(F;x) = (1+x) \bullet \left(1+3x+x^2\right) = 1+4x+4x^2+x^3$$

is palindromic and all its roots are real. Another example is $F = 3K_1$. □

By inspection, one can assert that there is no tree T with $\alpha(T) = 3$, such that $I(T;x)$ is palindromic (all the candidates are depicted in Figure 4).

Theorem 2.2. *For every integer α, with $2 \leq \alpha \neq 3$, there is a forest F consisting of at most two non-trivial trees, such that:*
(i) $\alpha(F) = \alpha$, *and*
(ii) $I(F;x)$ *is palindromic and all its roots are real.*

Proof. Let T_{real} be a tree on $q \geq 1$ vertices, whose independence polynomial $I(T_{\text{real}};x)$ has only real roots. For instance, one can choose $T_{\text{real}} = P_q$, and then, according to Theorem 1.2, $I(T_{\text{real}};x)$ has only real roots, since P_q is a claw-free graph.

Case 1. $\alpha = 2q, q \geq 1$.

The graph $F = T_{\text{real}} \circ 2K_1$ is a tree with $\alpha(F) = 2q$, and by Theorem 2.1, (ii), the polynomial $I(F;x)$ is palindromic and has only real roots.

Case 2. $\alpha = 2q+1, q \geq 2$.

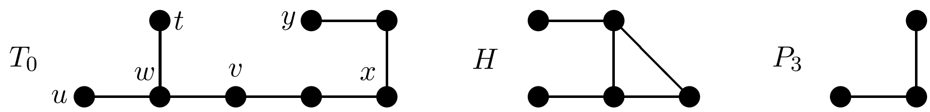

FIGURE 6. Two non-isomorphic graphs, namely, the tree T_0 and $G_0 = H \cup P_3$, that have the same independence polynomial

Firstly, the tree T_0 from Figure 6 has $\alpha(T_0) = 5$ and, using Proposition 1.1, (iii), we infer that

$$\begin{aligned} I(T_0;x) &= I(T_0 - v;x) + x \bullet I(T_0 - N[v];x) \\ &= I(P_3;x) \bullet I(P_4;x) + x \bullet (1+x)^2 \bullet I(P_3;x) \\ &= I(P_3;x) \bullet I(H;x) = \left(1+3x+x^2\right) \bullet \left(1+5x+5x^2+x^3\right) \\ &= \left(1+3x+x^2\right) \bullet \left(1+4x+x^2\right) \bullet (1+x) \\ &= 1+8x+21x^2+21x^3+8x^4+x^5. \end{aligned}$$

Consequently, the independence polynomial of T_0 is palindromic and has only real roots.

Further, the forest

$$F = T_0 \cup (T_{\text{real}} \circ 2K_1), q \geq 1,$$

has $\alpha(F) = 5+2q \geq 7$, and, according to Theorem 1.3, its independence polynomial

$$I(F;x) = I(T_0;x) \bullet I(T_{\text{real}} \circ 2K_1;x)$$

is palindromic, and clearly, all its roots are real. □

Remark 2.1. It was proved in [30] that if $G = (V,E)$ has a unique maximum stable set, say S, that satisfies $|S \cap N(v)| = 2$ for every $v \in V - S$, and $s_{\alpha(G)-1} = |V|$, then $I(G;x)$ is palindromic. The tree T_0 from Figure 6 shows that the converse of this assertion is not true, namely, $I(T_0;x)$ is palindromic, but there is $w \in V(T_0) - S$, such that $|S \cap N(w)| = 3$, where $S = \{u,t,v,x,y\}$ is the unique maximum stable set of T_0.

Remark 2.2. The forest $F = (P_q \circ 2K_1) \cup K_1$ has $\alpha(G) = 2q+1$ and

$$I(F;x) = (1+x) \bullet I(P_q \circ 2K_1;x).$$

According to Theorem 1.3, we get that $I(F;x)$ is palindromic, and by Theorem 2.1(ii), it follows that $I(F;x)$ has only real roots.

Recall that a *centipede* is a tree defined by $W_n = P_n \circ K_1, n \geq 1$ (see Figure 7). For example, $W_1 = K_2, W_2 = P_4$.

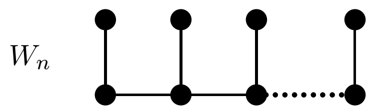

FIGURE 7. The centipede W_n

It was proved in [24] that $I(W_n;x) = (1+x)^n \cdot I(H;x)$, where H is a claw-free graph. Hence, W_n is a tree, whose $I(W_n;x)$ has only real roots, because, by Theorem 1.2, $I(H;x)$ has all the roots real. In other words, in the proof of Theorem 2.2, one can choose a centipede as T_{real}.

3. CONCLUSIONS AND OPEN PROBLEMS

In this paper we proved that for some non-trivial forests, their independence polynomials are palindromic and have all the roots real. By a well-known theorem of Newton, all these polynomials are log-concave, hence unimodal, so giving support to Conjecture 1.1.

The proof of Theorem 2.2 suggests the following problem.

Problem 3.1. *For every* **odd** *positive integer $\alpha > 5$, find a tree T such that $\alpha(T) = \alpha$, and $I(T;x)$ is palindromic and has only real roots.*

Notice that the graphs H_1, H_2 from Figure 1 are connected, $\alpha(H_1) = \alpha(H_2) = 3$, only one of them is bipartite, and their independence polynomials are palindromic and have only real roots.

Problem 3.2. *For every* **odd** *positive integer $\alpha > 3$, find a connected graph G different from a tree, such that $\alpha(G) = \alpha$, and $I(G;x)$ is palindromic and has only real roots.*

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