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ON THE CONVERGENCE AND ADHERENCE OF GRILLS

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Abstract. We define the convergence and adherence of grill in topological space with respect to the family of all α-open sets, semiopen sets, preopen sets, b-open sets and β-open sets, characterize such sets and discuss their properties.

1. INTRODUCTION AND PRELIMINARIES

Grills in a topological space (X, τ) is initiated by G. Choquet [6] in 1947. Properties of grill in topological spaces are further investicated in [8, 10, 11, 15, 16]. In this paper, we define the convergence and adherence of grill in the following family of sets obtained from τ, namely the family of all semiopen sets (σ), preopen sets (π), b-open sets (b) and β-open sets (β).

A *grill* $\mathcal{G}$ [6] is a nonempty collection of subsets of X which satisfies (i)$A \in \mathcal{G}$ and $A \subset B \subset X$ implies $B \in \mathcal{G}$ and (ii) A, $B \subset X$ and $A \cup B \in \mathcal{G}$ implies $A \in \mathcal{G}$ or $B \in \mathcal{G}$. By a space (X, τ), we always mean a topological space (X, τ) with no separation properties assumed. If $A \subset X$, $\mathrm{cl}(A)$ and $\mathrm{int}(A)$ will, respectively, denote the closure and interior of A in (X, τ). A subset A of a topological space is said to be semiopen [9] (resp. preopen [1], b-open [4], β-open [2] or semipreopen [3]) if $A \subset \mathrm{cl}(\mathrm{int}(A)$(resp. $A \subset \mathrm{int}(\mathrm{cl}(A)), A \subset \mathrm{int}(\mathrm{cl}(A)) \cup \mathrm{cl}(\mathrm{int}(A)), A \subset \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A))).$) A filter $\mathcal{F}$ on a space X is said to θ-adheres at $x \in X$ if for each $F \in \mathcal{F}$ and each open set U containing x, $F \cap \mathrm{cl}(U) \neq \emptyset$. $\mathcal{F}$ is said to θ-converge to $x \in X$ if for each open set U containing x, there corresponds $F \in \mathcal{F}$ such that $F \subset \mathrm{cl}(U)$. If open sets are replaced by preopen sets, we get $p(\theta)$-convergence and $p(\theta)$-adherence defined and discussed in [10]. If $\mathcal{G}$ is a grill (or filter) on a space X, then the *section of* $\mathcal{G}$, [10] denoted by $\sec \mathcal{G}$, is given by $\sec \mathcal{G} = \{A \subset X \mid A \cap G \neq \emptyset, \text{ for all } G \in \mathcal{G}\}$. Let (X, τ) be a space and $\mathcal{P} = \{\sigma, \pi, b, \beta\}$. For $\mu \in \mathcal{P}$ and a subset A of X, $i_\mu(A)$ is the union of all μ-open subsets of A and $c_\mu(A)$ is the intersection of the μ-closed supersets of A. Clearly, for $x \in X$ and $A \subset X$, $x \in c_\mu(A)$ if and only if $U \cap A \neq \emptyset$ for all $U \in \mu$ containing x. For any subset A of X, let us denote by $c_{\theta_\mu}(A)$, the

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set of all points $x \in X$ such that $c_\mu(U) \cap A \neq \emptyset$ whenever $x \in U \in \mu$. That is, $c_{\theta_\mu}(A) = \{x \in X \mid A \cap c_\mu(U) \neq \emptyset$ for every μ-open set U containing $x\}$. The following lemma will be useful in the sequel.

Lemma 1.1. [16] *Let (X, τ) be a space. Then the following hold.*
(a) *For any grill (resp. filter) $\mathcal{G}$ on a space X,* $\sec \mathcal{G}$ *is a filter (resp. grill) on X.*
(b) *If $\mathcal{F}$ is a filter and $\mathcal{G}$ is a grill such that $\mathcal{F} \subset \mathcal{G}$, then there is an ultrafilter $\mathcal{U}$ on X such that $\mathcal{F} \subset \mathcal{U} \subset \mathcal{G}$.*

2. MAIN THEOREMS

Let (X, τ) be a space and $\mathcal{P} = \{\sigma, \pi, b, \beta\}$. Throughout the paper, $\mu \in \mathcal{P}$. A grill $\mathcal{G}$ on X is said to be *θ_μ-adheres* at x in X if for each $U \in \mu$ containing x and each $G \in \mathcal{G}$, $c_\mu(U) \cap G \neq \emptyset$. $\mathcal{G}$ is said to *θ_μ-converges* to $x \in X$ if for each $U \in \mu$ containing x, there exists $G \in \mathcal{G}$ such that $G \subset c_\mu(U)$. Clearly, a grill $\mathcal{G}$ is θ_μ-converges to $x \in X$ if and only if $\mathcal{G}$ contains the collection $\{c_\mu(A) \mid x \in A \in \mu\}$. A filter $\mathcal{F}$ on a topological space (X, τ) is said to be *θ_μ-adheres* at $x \in X$ if for each $U \in \mu$ containing x and each $F \in \mathcal{F}$, $c_\mu(U) \cap F \neq \emptyset$. $\mathcal{F}$ is said to *θ_μ-converges* to $x \in X$ if for each $U \in \mu$ containing x, there exists $F \in \mathcal{F}$ such that $F \subset c_\mu(U)$. The following Theorem 2.1 shows that θ_μ-adherence of a grill is strictly stronger than that of θ_μ-convergence.

Theorem 2.1. *Let (X, τ) be a space with a grill $\mathcal{G}$. If $\mathcal{G}$ θ_μ-adheres at some point $x \in X$, then $\mathcal{G}$ is θ_μ-convergent to x.*

Proof. Suppose $\mathcal{G}$ θ_μ-adheres at $x \in X$. Then for each μ-open set U containing x and each $G \in \mathcal{G}$, $c_\mu(U) \cap G \neq \emptyset$. Now $c_\mu(U) \cap G \neq \emptyset$ implies that $c_\mu(U) \in \sec \mathcal{G}$ for each μ-open set U containing x. Moreover, $X - c_\mu(U) \notin \mathcal{G}$ whenever $x \in U \in \mu$. Since $X \in \mathcal{G}$ and $\mathcal{G}$ is a grill on X, $c_\mu(U) \in \mathcal{G}$ for each μ-open set containing x. Thus, $\mathcal{G}$ is θ_μ-convergent to x. □

If $\mu = \pi$ in the above Theorem 2.1, we get Theorem 2.6 of [10] and Example 2.7 of [10] shows that the converse of the above theorem is not true for $\mu = \pi$. Let (X, τ) be a space, $\mathcal{G}$ be a grill on X and $x \in X$. We define $\mathcal{G}(\theta_\mu, x) = \{A \subset X \mid x \in c_{\theta_\mu}(A)\}$ and $\sec \mathcal{G}(\theta_\mu, x) = \{A \subset X \mid A \cap G \neq \emptyset$ for all $G \in \mathcal{G}(\theta_\mu, x)\}$. The following Theorem 2.2 gives a characterization of θ_μ-adherence of a grill in terms of $\mathcal{G}(\theta_\mu, x)$ which is nothing but Theorem 2.9 of [10] if $\mu = \pi$ and Theorem 2.3 gives a characterization of θ_μ-convergence of a grill which is nothing but Theorem 2.10 of [10] if $\mu = \pi$.

Theorem 2.2. *Let $\mathcal{G}$ be a grill on a space (X, τ). Then the grill $\mathcal{G}$ θ_μ-adheres at $x \in X$ if and only if $\mathcal{G} \subset \mathcal{G}(\theta_\mu, x)$.*

Proof. $\mathcal{G}$ θ_μ-adheres at $x \in X$ implies that $c_\mu(U) \cap G \neq \emptyset$ for all $G \in \mathcal{G}$ and for every μ-open set U containing x. Therefore, $x \in c_{\theta_\mu}(G)$ for all $G \in \mathcal{G}$ and so $G \in \mathcal{G}(\theta_\mu, x)$ for all $G \in \mathcal{G}$ which implies that $\mathcal{G} \subset \mathcal{G}(\theta_\mu, x)$. Conversely, suppose that $\mathcal{G} \subset \mathcal{G}(\theta_\mu, x)$. Then for every $G \in \mathcal{G}$, $G \in \mathcal{G}(\theta_\mu, x)$ and so $x \in c_{\theta_\mu}(G)$ which implies that $c_\mu(U) \cap G \neq \emptyset$ for every $U \in \mu$ containing x. Hence $\mathcal{G}\,\theta_\mu$-adheres at the point x. □

Theorem 2.3. *Let $\mathcal{G}$ be a grill on a space (X, τ). Then $\mathcal{G}$ is θ_μ-convergent to $x \in X$ if and only if* $\sec \mathcal{G}(\theta_\mu, x) \subset \mathcal{G}$.

Proof. Let $\mathcal{G}$ be a grill on X which θ_μ-converges to $x \in X$. Then for each μ-open set U containing x, there exists a $G \in \mathcal{G}$ such that $G \subset c_\mu(U)$. Since $\mathcal{G}$ is a grill on X, $c_\mu(U) \in \mathcal{G}$. Let $A \in \sec \mathcal{G}(\theta_\mu, x)$. Then $A \cap G \neq \emptyset$ for all $G \in \mathcal{G}(\theta_\mu, x)$ and so $X - A \notin \mathcal{G}(\theta_\mu, x)$. Now $X - A \notin \mathcal{G}(\theta_\mu, x)$ implies that $x \notin c_{\theta_\mu}(X - A)$ and so $c_\mu(V) \cap (X - A) = \emptyset$ for some $V \in \mu$ containing x. Again, $c_\mu(V) \cap (X - A) = \emptyset$ implies that $c_\mu(V) \subset A$ which in turn implies that $A \in \mathcal{G}$. Thus, $\sec(\theta_\mu, x) \subset \mathcal{G}$. Conversely, suppose that $\mathcal{G}$ does not θ_μ-converge to $x \in X$. Then there exists a μ-open set U containing x such that $c_\mu(U) \notin \mathcal{G}$ and so $c_\mu(U) \notin \sec \mathcal{G}(\theta_\mu, x)$. Since $c_\mu(U) \notin \sec\, \mathcal{G}(\theta_\mu,\ x)$, there exists $G \in \mathcal{G}(\theta_\mu,\ x)$ such that $c_\mu(U) \cap G = \emptyset$. Since $G \in \mathcal{G}(\theta_\mu,\ x)$, $x \in c_{\theta_\mu}(G)$ and so $c_\mu(U) \cap G \neq \emptyset$ for all μ-open set U containing x, a contradiction. Hence $\mathcal{G}$ θ_μ-converges to x. □

A nonempty set A of a space (X, τ) is said to be *μ-closed* relative to X if for every cover $\{V_\alpha \mid \alpha \in \Delta\}$ of A by μ-open sets of X, there exists a finite subset Δ_0 of Δ such that $A \subset \cup\{c_\mu(V_\alpha) \mid \alpha \in \Delta_0\}$. A space X is μ-closed if $A = X$. If $\mu = \tau$, then the family of all τ-closed spaces coincides with the family of all almost compact(QHC) spaces [14], if $\mu = \pi$, then the family of all π-closed spaces coincides with the family of all p-closed spaces [1, 10], if $\mu = \beta$, then we have the family of all β-closed spaces [5], if $\mu = b$, then we have the family of all b-closed spaces [13] and if $\mu = \sigma$, then the family of all σ-closed spaces coincides with the family of all s-closed spaces [7]. The following Theorem 2.4 characterizes μ-closed spaces, which is also true for $\mu = \tau$.

Theorem 2.4. *Let (X, τ) be a space and $\mu \in \mathcal{P}$. Then the following are equivalent.*
(a) *X is μ-closed.*
(b) *Every maximal filterbase θ_μ-converges to some point of X.*
(c) *Every filterbase θ_μ-adheres at some point of X.*
(d) *For every family $\{V_\alpha \mid \alpha \in \Delta\}$ of μ-closed subsets such that $\cap\{V_\alpha \mid \alpha \in \Delta\} = \emptyset$, there exists a finite subset Δ_0 of Δ such that $\cap\{i_\mu(V_\alpha) \mid \alpha \in \Delta_0\} = \emptyset$.*

Proof. (a)⇒(b). Let $\mathcal{F}$ be a maximal filterbase on X. Suppose that $\mathcal{F}$ does not θ_μ-converge to any point of X. Since $\mathcal{F}$ is maximal, $\mathcal{F}$ does not θ_μ-adheres at any point of X. For each $x \in X$, there exists $F_x \in \mathcal{F}$ and μ-open set U_x containing x such that $F_x \cap c_\mu(U_x) = \emptyset$. Now $\{U_x \mid x \in X\}$ is a μ-open cover for X. By (a), there exists a finite subset $\{x_1, x_2, ..., x_n\}$ of X such that $X = \cup\{c_\mu(U_{x_i}) \mid 1 \leq i \leq n\}$. Since $\mathcal{F}$ is a filterbase, there exists $F \in \mathcal{F}$ such that $F \subset \cap F_{x_i}$. Now, $\cap F_{x_i} = (\cap F_{x_i}) \cap (\cup c_\mu(U_{x_i})) = \cup((\cap F_{x_i}) \cap c_\mu(U_{x_i})) = \emptyset$. Hence $F = \emptyset$, a contradiction.

(b)$\Rightarrow$(c). Let $\mathcal{F}$ be a filterbase on X. Then there exists a maximal filterbase $\mathcal{F}_0$ such that $\mathcal{F} \subset \mathcal{F}_0$. By (b), $\mathcal{F}_0$ θ_μ-converges to some point $x \in X$. For every $F \in \mathcal{F}$ and every $U \in \mu$ containing x, there exists $F_0 \in \mathcal{F}_0$ such that $F_0 \subset c_\mu(U)$. Now $F_0 \subset c_\mu(U)$ implies that $F_0 \cap F \subset c_\mu(U) \cap F$. Since $\mathcal{F}_0$ is a filterbase, $F \cap F_0 \neq \emptyset$ and so $F \cap c_\mu(U) \neq \emptyset$. Hence $\mathcal{F}$ θ_μ-adheres at x.

(c)$\Rightarrow$(d). Let $\{V_\alpha \mid \alpha \in \Delta\}$ be a family of μ-closed subsets of X such that $\cap\{V_\alpha \mid \alpha \in \Delta\} = \emptyset$. Let $\mathcal{I}$ be the family of all finite subsets of Δ. Assume that $A_I = \cap\{i_\mu(V_\alpha) \mid \alpha \in I\} \neq \emptyset$ for every $I \in \mathcal{I}$. Then the family $\mathcal{F} = \{A_I \mid I \in \mathcal{I}\}$ is a filterbase on X. By hypothesis, $\mathcal{F}$ θ_μ-adheres at some point $x \in X$. Since $\{X - V_\alpha \mid \alpha \in \Delta\}$ is a μ-open cover of X, $x \in X - V_\beta$ for some $\beta \in \Delta$. Since $c_\mu(X - V_\beta) \cap i_\mu(V_\beta) = \emptyset$, we have a contradiction to the fact that $\mathcal{F}$ θ_μ-adheres at $x \in X$. Thus $\cap\{i_\mu(V_\alpha) \mid \alpha \in I\} = \emptyset$ for some $I \in \mathcal{I}$.

(d)$\Rightarrow$(a). Let $\{V_\alpha \mid \alpha \in \Delta\}$ be a cover of X by μ-open sets of X. Then $\{X - V_\alpha \mid \alpha \in \Delta\}$ is a family of μ-closed subsets of X such that $\cap\{X - V_\alpha \mid \alpha \in \Delta\} = \emptyset$. By (d), there exists a finite subset Δ_0 of Δ such that $\cap\{i_\mu(X - V_\alpha) \mid \alpha \in \Delta_0\} = \emptyset$. Hence $X - \cap\{i_\mu(X - V_\alpha) \mid \alpha \in \Delta_0\} = X$ which implies that $\cup\{c_\mu(V_\alpha) \mid \alpha \in \Delta_0\} = X$. Hence X is μ-closed. □

The following Theorem 2.5 gives a characterization of μ-closed spaces in terms of θ_μ-convergence of a grill. Theorem 2.6 gives a characterization of μ-closed relative to X subsets in terms of θ_μ-convergence of a grill. Theorem 3.4 of [10] gives similar characterization of π-closed relative to X subsets. The following Corollary 2.1 shows that Theorem 2.5 is true for $\mu = \tau$.

Theorem 2.5. *A space (X, τ) is μ-closed if and only if every grill $\mathcal{G}$ on X θ_μ-converges in X.*

Proof. Let $\mathcal{G}$ be a grill in a μ-closed space X. Then by Lemma 1.1, $\sec \mathcal{G}$ is a filter on X. Let $B \in \sec \mathcal{G}$. Then $B \cap G \neq \emptyset$ for every $G \in \mathcal{G}$ which implies that $X - B \notin \mathcal{G}$. Hence $B \in \mathcal{G}$, since $\mathcal{G}$ is a grill. Thus, $\sec \mathcal{G} \subset \mathcal{G}$. By Lemma 1.1, there is an ultrafilter $\mathcal{U}$ on X such that $\sec \mathcal{G} \subset \mathcal{U} \subset \mathcal{G}$. Since X is μ-closed, by Theorem 2.4, $\mathcal{U}$ θ_μ-converges to some point $x \in X$. Then for each μ-open set U containing x, there exists some $G \in \mathcal{U}$ such that $G \subset c_\mu(U)$. Since $c_\mu(U) \in \mathcal{U}$, $c_\mu(U) \in \mathcal{G}$ for every $U \in \mu$ containing x. Hence $\mathcal{G}$ θ_μ-converges to x. Conversely, let $\mathcal{U}$ be an ultrafilter on X. Since every ultrafilter is a grill, $\mathcal{U}$ θ_μ-converges to some point of X. □

Corollary 2.1. *A space (X, τ) is almost compact if and only if every grill $\mathcal{G}$ on X θ-converges in X.*

Corollary 2.2. *A space (X,τ) is s-closed if and only if every grill $\mathcal{G}$ on X θ_σ-converges in X.*

Corollary 2.3. [10, Theorem 3.3] *A space (X,τ) is p-closed if and only if every grill $\mathcal{G}$ on X θ_π-converges in X.*

Corollary 2.4. *A space (X,τ) is b-closed if and only if every grill $\mathcal{G}$ on X θ_b-converges in X.*

Corollary 2.5. *A space (X,τ) is β-closed if and only if every grill $\mathcal{G}$ on X θ_β-converges in X.*

Theorem 2.6. *A subset A of a space (X,τ) is μ-closed relative to X if and only if every grill $\mathcal{G}$ on X with $A\in\mathcal{G}$, θ_μ-converges to a point in A.*

Proof. Let A be μ-closed relative to X and $\mathcal{G}$ be a grill on X such that $A\in\mathcal{G}$ and $\mathcal{G}$ does not θ_μ-converge to any point of A. Then, for each $x\in A$, there exists $U_x\in\mu$ containing x such that $c_\mu(U_x)\notin\mathcal{G}$. Now, $\{U_x \mid x\in A\}$ is a cover of A by μ-open sets of X. Since A is μ-closed relative to X, there exists a finite subset B of A such that $A\subset\cup\{c_\mu(U_x)\mid x\in B\}$. Since $\mathcal{G}$ is a grill, $\cup\{c_\mu(U_x)\mid x\in B\}\notin\mathcal{G}$. Hence $A\notin\mathcal{G}$, a contradiction. Hence $\mathcal{G}$ θ_μ-converges to some point of A. Conversely, let $\mathcal{U}=\{U_\alpha\mid\alpha\in\Delta\}$ be a cover of A by μ-open sets of X. If A is not μ-closed relative to X, then for every finite subset Δ_0 of Δ, $A-\cup\{c_\mu(U_\alpha)\mid\alpha\in\Delta_0\}\neq\emptyset$. Let $\mathcal{F}=\{A-\cup c_\mu(U_\alpha)\mid\alpha\in\Delta_0,\Delta_0\subset\Delta$ is finite$\}$. Then $\mathcal{F}$ is a filterbase on X. The family $\mathcal{F}$ can be extended to an ultrafilter $\mathcal{M}$ on X. Then $\mathcal{M}$ is a grill on X. Since for every $F\in\mathcal{F}$, $F\subset A$, we have $A\in\mathcal{M}$. Since $\mathcal{U}$ is a cover of A, for each $x\in A$, there exists $\beta\in\Delta$ such that $x\in U_\beta$. For any $G\in\mathcal{M}$, $G\cap(A-c_\mu(U_\beta))\neq\emptyset$. Hence $G\not\subseteq c_\mu(U_\beta)$ for all $G\in\mathcal{M}$ and so $\mathcal{M}$ does not θ_μ-converge to any point of A, a contradiction. □

Theorem 2.7. *Let (X,τ) be a space such that every grill $\mathcal{G}$ on X with the property $\cap\{c_{\theta_\mu}(G_i)\mid 1\leq i\leq n\}\neq\emptyset$ for every finite subfamily $\{G_1,G_2,...,G_n\}$ of $\mathcal{G}$, θ_μ-adheres in X, then X is a μ-closed space.*

Proof. Let $\mathcal{U}$ be any ultrafilter on X. Then $\mathcal{U}$ is a grill on X. Also, for each finite subcollection $\{U_1,U_2,...,U_n\}$ of $\mathcal{U}$, $\cap\{U_i\mid 1\leq i\leq n\}\neq\emptyset$. Since

$$\cap\{U_i\mid 1\leq i\leq n\}\subset\cap\{c_{\theta_\mu}(U_i)\mid 1\leq i\leq n\},\quad \cap\{c_{\theta_\mu}(U_i)\mid 1\leq i\leq n\}\neq\emptyset.$$

Hence by hypothesis, $\mathcal{U}$ θ_μ-adheres in X. By Theorem 2.4, X is μ-closed. □

A grill $\mathcal{G}$ on X is said to be θ_μ-*linked* if for any two members $A,B\in\mathcal{G}$, $c_{\theta_\mu}(A)\cap c_{\theta_\mu}(B)\neq\emptyset$. $\mathcal{G}$ is said to be θ_μ-*conjoint* if for every finite subfamily $\{A_1,A_2,...,A_n\}$ of $\mathcal{G}$, $i_\mu(\cap\{c_{\theta_\mu}(A_i)\mid 1\leq i\leq n\})\neq\emptyset$. Clearly, every θ_μ-conjoint grill is θ_μ-linked. If $\mu=\pi$, we have the $p(\theta)$-linked grill [10] and $p(\theta)$-conjoint grill

[10]. Example 2.12 of [10] gave an example of a $p(\theta)$-linked grill which is not a $p(\theta)$-conjoint grill.

Theorem 2.8. *Let (X, τ) be a μ-closed space. Then every θ_μ-conjoint grill θ_μ-adheres in X.*

Proof. Let $\mathcal{G}$ be a θ_μ-conjoint grill on a μ-closed space X. Since $c_{\theta_\mu}(A)$ is μ-closed for every $A \subset X$, $\{c_{\theta_\mu}(A) \mid A \in \mathcal{G}\}$ is a collection of μ-closed sets in X. Since $\mathcal{G}$ is θ_μ-conjoint, for any finite subfamily $\{A_1, A_2, ..., A_n\}$ of $\mathcal{G}$, $i_\mu(\cap\{c_{\theta_\mu}(A_i) \,|\, 1 \leq i \leq n\}) \neq \emptyset$ and so $\cap\{i_\mu(c_{\theta_\mu}(A_i)) \,|\, 1 \leq i \leq n\} \neq \emptyset$. Hence by Theorem 2.4, $\cap\{c_{\theta_\mu}(A) \,|\, A \in \mathcal{G}\} \neq \emptyset$ and so there exists $x \in X$ such that $x \in c_{\theta_\mu}(A)$ for every $A \in \mathcal{G}$ which implies that $A \in \mathcal{G}(\theta_\mu, x)$ for all $A \in \mathcal{G}$ which in turn implies that $\mathcal{G} \subset \mathcal{G}(\theta_\mu, x)$. Therefore, $\mathcal{G}$ θ_μ-adheres at x, by Theorem 2.2. □

The following Lemma 2.1 will be useful to prove Theorem 2.9 below.

Lemma 2.1. *Let (X, τ) be a space and $\mu \in \{\sigma, b, \beta\}$ and $A \subset X$. Then $c_{\theta_\mu}(c_{\theta_\mu}(A)) = c_{\theta_\mu}(A)$.*

Proof. For $\mu = \beta$, the proof is in [12]. For $\mu = b$, the proof is in [13]. For $\mu = \sigma$, the proof is as follows. Clearly, $c_{\theta_\mu}(A) \subset c_{\theta_\mu}(c_{\theta_\mu}(A))$. If $x \notin c_{\theta_\mu}(A)$, then there exists a semiregular set V containing x such that $V \cap A = \emptyset$. Since V is semiregular, $V \cap c_{\theta_\mu}(A) = \emptyset$ and so $x \notin c_{\theta_\mu}(c_{\theta_\mu}(A))$. □

Theorem 2.9. *Let (X, τ) be a μ-closed space where $\mu \in \{\sigma, b, \beta\}$. Then every grill $\mathcal{G}$ on X, with the property that $\cap\{c_{\theta_\mu}(G_i) \mid 1 \leq i \leq n\} \neq \emptyset$ for every finite subfamily $\{G_1, G_2, \ldots, G_n\}$ of $\mathcal{G}$, θ_μ-adheres in X.*

Proof. Let X be a μ-closed space. $\mathcal{G} = \{G_\alpha \mid \alpha \in \Delta\}$ be a grill on X with the property $\cap\{c_{\theta_\mu}(G_\alpha) \mid \alpha \in \Delta_0\} \neq \emptyset$ for every finite subset Δ_0 of Δ. Consider the family $\mathcal{F} = \{\cap\{c_{\theta_\mu}(G_\alpha) \mid \alpha \in \Delta_0\} \mid \Delta_0 \subset \Delta \text{ is finite}\}$. Then $\mathcal{F}$ is a filterbase on X. Since X is μ-closed, $\mathcal{F}$ θ_μ-adheres at some $x \in X$ which implies that $x \in c_{\theta_\mu}(c_{\theta_\mu}(G))$ for every $G \in \mathcal{G}$ which in turn implies that $x \in c_{\theta_\mu}(G)$ for every $G \in \mathcal{G}$. Hence $\mathcal{G} \subset \mathcal{G}(\theta_\mu, x)$ and so the proof follows from Theorem 2.2. □

Corollary 2.6. *Let (X, τ) be a space where $\mu \in \{\sigma, b, \beta\}$. Then the following are equivalent.*
(a) *Every grill $\mathcal{G}$ on X, with the property that $\cap\{c_{\theta_\mu}(G_i) \mid 1 \leq i \leq n\} \neq \emptyset$ for every finite subfamily $\{G_1, G_2, ..., G_n\}$ of $\mathcal{G}$, θ_μ-adheres in X.*
(b) *X is μ-closed.*

Proof. (a)⇒(b). The proof follows from Theorem 2.7.
(b)⇒(a). The proof follows from Theorem 2.9. □

A space X is μ *compact* where $\mu \in \{\tau, \alpha, \sigma, \pi, b, \beta\}$ if every cover $\mathcal{U}$ of X by μ-open sets of X has a finite subcover. Every μ-compact space is μ-closed. A space X is μ *regular* if for each $x \in X$ and each $U \in \mu$ containing x, there exists a μ-open set V containing x such that $c_\mu(V) \subset U$. A space X is θ_μ*-regular* if every grill on X which θ_μ-converges must μ-converge. A grill $\mathcal{G}$ on X is said to be μ*-converge* to a point $x \in X$ if $\mu(x) \subset \mathcal{G}$, where $\mu(x)$ denote the family of all μ-open sets containing x.

Theorem 2.10. *A space (X, τ) is μ-compact if and only if every grill μ-converges.*

Proof. Let $\mathcal{G}$ be a grill on a μ-compact space X such that $\mathcal{G}$ does not μ-converge to any point $x \in X$. Then for each $x \in X$, there exists a μ-open set U_x containing x with $U_x \notin \mathcal{G}$. Since $\{U_x \mid x \in X\}$ is a cover of the μ-compact X by μ-open sets, there exist finitely many points $\{x_1, x_2, ..., x_n\}$ in X such that $X = \cup\{U_{x_i} \mid 1 \le i \le n\}$. Since $X \in \mathcal{G}$, there exists U_{x_i} such that $U_{x_i} \in \mathcal{G}$ for $1 \le i \le n$, a contradiction. Conversely, let every grill on X μ-converge. Suppose X is not μ-compact. Then there exists a cover $\mathcal{U}$ of X by μ-open sets of X having no finite subcover. Then $\mathcal{F} = \{X - \cup\mathcal{U}_0 \mid \mathcal{U}_0$ is a finite subcollection of $\mathcal{U}\}$ is a filterbase on X. Then $\mathcal{F}$ is contained in an ultrafilter $\mathcal{G}$ and so $\mathcal{G}$ is a grill on X. By hypothesis, $\mathcal{G}$ μ-converges to some $x \in X$. Then for some $U \in \mathcal{U}$, $x \in U$ and hence $U \in \mathcal{G}$. But $X - U \in \mathcal{F} \subset \mathcal{G}$. Hence U and $X - U$ both belong to $\mathcal{G}$, which is an ultrafilter, a contradiction. □

Theorem 2.11. *A μ-compact space X is μ-closed. The converse holds if X is θ_μ-regular.*

Proof. Suppose $\mathcal{G}$ is a grill on a μ-closed space X. Then $\mathcal{G}$ θ_μ-converges in X, by Theorem 2.5. Since X is θ_μ-regular, $\mathcal{G}$ μ-converges. By Theorem 2.10, X is μ-compact. □

Theorem 2.12. *Every μ-regular space is θ_μ-regular.*

Proof. Let $\mathcal{G}$ be a grill on a μ-regular space X. Suppose $\mathcal{G}$ θ_μ-converges to a point $x \in X$. For each $U \in \mu$ containing x, there exists a $V \in \mu$ containing x such that $c_\mu(V) \subset U$. By hypothesis, $c_\mu(V) \in \mathcal{G}$ and so $U \in \mathcal{G}$. Hence $\mathcal{G}$ μ-converges to x. Thus, X is θ_μ-regular. □

REFERENCES

[1] Abd El-Aziz Abo-Khadra: On generalized forms of compactness, Masters Thesis, Faculty of Science, Tanta University, Egypt, 1989.

[2] M. E. Abd El-Monsef, S. N. El-Deeb and R. A. Mahmoud: *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assiut Univ., **12**(1983), No. 1, 77-90.

[3] D. Andrijević: *Semi-preopen sets*, Mat. Vesn., **38**(1986), 24-32.

[4] D. Andrijević: *On b-open sets*, Mat. Vesn., **48**(1996), 59-64.

[5] C. K. Basu and M. K. Ghosh: *β-closed spaces and β-θ-subclosed graphs*, Eur. J. Pure Appl. Math., **1**(2008), No. 3, 40-50.

[6] G. Choquet: *Sur les notions de Filtre et grille*, C. R. Math., Acad. Sci. Paris, **224**(1947), 171-173.

[7] G. Di Maio and T. Noiri: *On s-closed spaces*, Indian J. Pure Appl. Math., **18**(1987), No. 3, 226-233.

[8] E. Hatir and S. Jafari: *On some new classes of sets and a new decomposition of continuity via grills*, J. Adv. Math. Stud., **3**(2010), No. 1, 33-40.

[9] N. Levine: *Semi-open sets and semi-continuity in topological spaces*, Amer. Math. Monthly, **70**(1963), 36-41.

[10] M. N. Mukherjee and B. Roy: *p-closed topological spaces in terms of grills*, Hacet. J. Math. Stat., **35**(2006), No. 2, 147-154.

[11] M. N. Mukherjee, B. Roy and P. Sinha: *Concerning p-closed topological spaces*, Revista De La Academia Canaria De Ciencias, XIV(2002), No. 1-2, 9-23.

[12] T. Noiri: *Weak and strong forms of β-irresolute functions*, Acta Math. Hung., **99**(2003), No. 4, 315-328.

[13] J. H. Park: *Strongly θ-b-continuous functions*, Acta Math. Hung., **110**(2006), No. 4, 347-359.

[14] J. Porter and J. Thomas: On H-closed and minimal Hausdorff spaces, Trans. Amer. Math. Soc., **138**(1961), 159-170.

[15] B. Roy and M. N. Mukherjee: *On a type of compactness via grills*, Mat. Vesn., **59**(2007), 113-120.

[16] W. J. Thron: *Proximity structure and grills*, Math. Ann., **206**(1973), 35-62.

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