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AN IMPLICIT ITERATION METHOD FOR A FINITE FAMILY OF PSEUDOCONTRACTIVE MAPPINGS

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ABSTRACT. Strong convergence theorems for approximation of common fixed points of pseudocontractive mappings are proved in Banach spaces using an implicit iteration scheme. The results improve and extend the corresponding results of Osilike [5], Xu and Ori [9], Chidume and Shahzad [2] and others.

1. INTRODUCTION

Let E be a real Banach space and let J denote the normalized duality mapping from E into 2^{E^*} given by $J(x) = \{f \in E^*\colon \langle x, f\rangle = \|x\|^2 = \|f\|^2\}$, where E^* denotes the dual space of E and $\langle\cdot,\cdot\rangle$ denotes the generalized duality pairing. If E^* is strictly convex, then J is single-valued. In the sequel, we shall denote the single-value duality mapping by j.

Let K be a closed convex subset of E. Recall that a self-mapping $f\colon K \to K$ is a contraction on K if there exists a constant $\alpha \in (0,1)$ such that

$$\|f(x) - f(y)\| \leq \alpha\|x - y\|, \quad \text{for all } x, y \in K.$$

We use Π_K to denote the collection of all contractions on K. That is,

$$\Pi_K = \{f \mid f\colon K \to K \textit{ a contraction}\}.$$

A mapping T with domain $D(T)$ and $R(T)$ in E is called pseudocontractive if, for all $x, y \in D(T)$, there exists $j(x-y) \in J(x-y)$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x - y\|^2.$$

Numerous papers have been written on fixed point problems of nonlinear mappings. Related work can be found in [1]-[9]. Recently, Xu and Ori [9] have introduced an implicit iteration process for a finite family of nonexpansive mappings. Let $T_1, T_2, \ldots, T_N$ be N self-mappings of E and suppose that $F = \bigcap_{i=1}^{N} \neq \emptyset$, the set of common fixed points of $T_i, i = 1, 2, \ldots, N$. An implicit iteration process for a finite

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family of nonexpansive mappings is defined as follows with $\{t_n\}$ a real sequence in $(0,1)$, $x_0 \in E$:

$$\begin{aligned} x_1 &= t_1 x_0 + (1-t_1)T_1 x_1, \\ x_2 &= t_2 x_1 + (1-t_2)T_2 x_2, \\ &\vdots \\ x_N &= t_N x_{N-1} + (1-t_N)T_N x_N, \\ x_{N+1} &= t_{N+1} x_N + (1-t_{N+1})T_1 x_{N+1}, \\ &\vdots \end{aligned}$$

which can be written in the following compact form

$$x_n = t_n x_{n-1} + (1-t_n)T_n x_n, \ n \geq 1, \tag{1.1}$$

where $T_n = T_{n \bmod N}$.

Xu and Ori proved the weak convergence of this process to a common fixed point of the finite family defined in a Hilbert space. They further remarked that it is yet unclear what assumptions on the mapping and/or the parameters $\{t_n\}$ are sufficient to guarantee the strong convergence of the sequence $\{x_n\}$.

In 2004, Osilike [5] first extended Xu and Ori [9] from the class of nonexpansive maps to the more general class of strictly pseudocontractive maps in Hilbert space. He proved the following convergence theorems.

Theorem 1.1. *Let H be a real Hilbert space and let K be a nonempty closed convex subset of H. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K such that $F = \bigcap_{i=1}^N F(T_i) \neq \emptyset$. Let $x_0 \in K$ and let $\{\alpha_n\}_{n=1}^\infty$ be a sequence in $(0,1)$ such that $\lim_{n\to\infty} \alpha_n = 0$. Then the sequence $\{x_n\}_{n=1}^\infty$ defined by*

$$x_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n x_n, \quad n \geq 1,$$

where $T_n = T_{n \bmod N}$, converges weakly to a common fixed point of the mappings $\{T_i\}_{i=1}^\infty$.

Theorem 1.2. *Let E be a real Banach space and let K be a nonempty closed convex subset of E. Let $\{T_i\}_{i=1}^N$ be N strictly pseudocontractive self-maps of K such that $F = \bigcap_{i=1}^N F(T_i) \neq \emptyset$, and let $\{\alpha_n\}_{n=1}^\infty$ be a real sequence satisfying the conditions:*

$$\text{(i) } 0 < \alpha_n < 1; \quad \text{(ii) } \sum_{n=1}^\infty (1-\alpha_n) = \infty; \quad \text{(iii) } \sum_{n=1}^\infty (1-\alpha_n)^2 < \infty.$$

Let $x_0 \in K$ and let $\{x_n\}_{n=1}^\infty$ be defined by

$$x_n = \alpha_n = \alpha_n x_{n-1} + (1-\alpha_n)T_n x_n, \quad n \geq 1,$$

where $T_n = T_{n \bmod N}$. *Then* $\{x_n\}$ *converges strongly to a common fixed point of the mappings* $\{T_i\}_{i=1}^N$ *if and only if* $\liminf\limits_{n\to\infty} d(x_n, F) = 0$.

Remark 1.1. We note that Theorem 1.1 has only weak convergence even in a Hilbert space and Theorem 1.2 has strong convergence but imposed condition $\liminf\limits_{n\to\infty} d(x_n, F) = 0$.

In 2005, Chidume and Shahzad [2] proved the strong convergence of an implicit iteration process (1.1) to a common fixed point for a finite family of nonexpansive mappings. They proved the following theorem.

Theorem 1.3. *Let* E *be a uniformly convex Banach space,* K *be a nonempty closed convex subset of* E. *Let* $\{T_i\}_{i=1}^N$ *be* N *nonexpansive self-mappings of* K *with* $F = \bigcap\limits_{i=1}^{N} F(T_i) \neq \emptyset$. *Suppose that one of the mappings in* $\{T_i\}_{i=1}^N$ *is semi-compact. Let* $\{t_n\} \subset [\delta, 1-\delta]$ *for some* $\delta \in (0,1)$. *From arbitrary* $x_0 \in K$, *define the sequence* $\{x_n\}$ *by* (1.1). *Then* $\{x_n\}$ *converges strongly to a common fixed point of the mappings* $\{T_i\}_{i=1}^N$.

Remark 1.2. Chidume and Shahzad given an affirmative response to the question raised by Xu and Ori [9] but they imposed compactness condition on mappings.

In this paper, we will extend the process (1.1) to a process for finite family of pseudocontractive mappings with $\{\alpha_n\}$ and $\{\beta_n\}$ two real sequences in $(0,1)$, and an initial point $x_0 \in K$, which is defined as follows:

$$\begin{aligned}
y_1 &= \beta_1 f(x_0) + (1-\beta_1)x_0,\\
x_1 &= \alpha_1 y_1 + (1-\alpha_1)T_1 x_1,\\
y_2 &= \beta_2 f(x_1) + (1-\beta_2)x_1,\\
x_2 &= \alpha_2 y_2 + (1-\alpha_2)T_2 x_2,\\
&\ \vdots\\
y_N &= \beta_N f(x_{N-1}) + (1-\beta_N)x_{N-1},\\
x_N &= \alpha_N y_N + (1-\alpha_N)T_N x_N,\\
y_{N+1} &= \beta_{N+1} f(x_N) + (1-\beta_{N+1})x_N,\\
x_{N+1} &= \alpha_{N+1} y_{N+1} + (1-\alpha_{N+1})T_1 x_{N+1},\\
&\ \vdots
\end{aligned}$$

The scheme is expressed in a compact form as

$$\begin{aligned}
y_n &= \beta_n f(x_{n-1}) + (1-\beta_n)x_{n-1}, \quad n \geq 1,\\
x_n &= \alpha_n y_n + (1-\alpha_n)T_n x_n,
\end{aligned} \tag{1.2}$$

where $T_n = T_{n \bmod N}$.

Our purpose in this paper is to study the implicit iteration process (1.2) in the general setting of a uniformly smooth Banach space and prove the strong convergence of the process to a common fixed point of the mappings $\{T_i\}_{i=1}^{\infty}$. The results presented in this paper generalize and extend the corresponding results of Chidume and Shahzad [2], Osilike [5], Xu and Ori [9] and others.

2. PRELIMINARIES

Let E be a Banach space. Recall the norm of E is said to be Gateaux differentiable (and E is said to be smooth) if

$$\lim_{t \to 0} \frac{\|x + ty\| - \|x\|}{t} \tag{2.1}$$

exists for each x, y in its unit sphere $U = \{x \in E \,|\, \|x\| = 1\}$. It is said to be uniformly Frechet differentiable (and E is said to be uniformly smooth) if the limit in (2.1) is attained uniformly for $(x, y) \in U \times U$. It is well-known that a Banach space E is uniformly smooth if and only if the duality map J is single-valued and norm-to-norm uniformly continuous on bounded sets of E.

Recall that if C and D are nonempty subsets of a Banach space E such that C is nonempty closed convex and $D \subset C$, then a map $Q\colon C \to D$ is called a retraction from C onto D provided $Q(x) = x$ for all $x \in D$. A traction $Q\colon C \to D$ is sunny provided $Q(x + t(x - Q(x))) = Q(x)$ for all $x \in C$ and $t \geq 0$ whenever $x + t(x - Q(x)) \in C$. A sunny nonexpansive retraction is a sunny retraction, which is also nonexpansive.

We need the following lemmas for proof of our main results.

Lemma 2.1 ([8]). *Let E be a uniformly smooth Banach space, K a closed convex subset of E, $T\colon K \to K$ be a nonexpansive with $Fix(T) \neq \emptyset$ and $f \in \Pi_K$. For each $f \in \Pi_K$ and every $t \in 0, 1)$, then $\{x_t\}$ defined by*

$$x_t = tf(x_t) + (1 - t)Tx_t \tag{2.2}$$

converges strongly as $t \to 0$ to a fixed point of T.

In particular, if $f = u \in K$ is a constant, then (2.2) is reduced to the sunny nonexpansive retraction of Reich from K onto $Fix(T)$,

$$\langle Q(u) - u, J(Q(u) - p)\rangle \leq 0,\ u \in K, \quad p \in Fix(T).$$

Lemma 2.2 (Reich [6]). *Let E be a real uniformly smooth Banach space. Then there exists a nondecreasing continuous function $b\colon [0, \infty) \to [0, \infty)$ satisfying:*

(i) $b(ct) \leq cb(t)$, *for all* $c \geq 1$; (ii) $\lim_{t \to 0+} b(t) = 0$;

(iii) $\|x + y\|^2 \leq \|x\|^2 + 2\langle y, j(x)\rangle + \max\{\|x\|, 1\}\|y\|b(\|y\|)$, *for all* $x, y \in x$.

The inequality (iii) is called *Reich's inequality.*

Lemma 2.3 ([7]). *Let $\{a_n\}_{n=0}^{\infty}$ be a sequences of nonnegative real numbers satisfying the property*

$$a_{n+1} \leq (1 - \gamma_n)a_n + \gamma_n\sigma_n,\ n \geq 0,$$

where $\{\gamma_n\}_{n=0}^{\infty} \subset (0,1)$ *and* $\{\sigma_n\}_{n=0}^{\infty}$ *are such that:*

(i) $\lim_{n\to\infty} \gamma_n = 0$ *and* $\sum_{n=0}^{\infty} \gamma_n = \infty$; (ii) *either* $\limsup_{n\to\infty} \sigma_n \le 0$ *or* $\sum_{n=0}^{\infty} |\gamma_n \sigma_n| < \infty$.

Then $\{a_n\}_{n=0}^{\infty}$ *converges to* 0.

3. MAIN RESULTS

Theorem 3.1. *Let* E *be a uniformly smooth Banach space and let* K *be a nonempty closed convex subset of* E. *Let* $\{T_i\}_{i=1}^{N}$ *be* N *pseudocontractive self-maps of* K *such that* $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$. *Let* $f \in \Pi_K$, $x_0 \in K$ *and* $\{\alpha_n\}_{n=1}^{\infty}, \{\beta_n\}_{n=1}^{\infty}$ *be two sequences in* $(0,1)$ *satisfying the following conditions:*

(i) $\alpha_n \to 0, \beta_n \to 0, n \to \infty$; (ii) $\sum_{n=0}^{\infty} \beta_n = \infty$.

Then the sequence $\{x_n\}$ *defined by* (1.2) *converges strongly to a common fixed point of the mappings* $\{T_i\}_{i=1}^{N}$.

Proof. First, we take a common fixed point $p \in F$, noting that

$$\begin{aligned} \|x_n - p\|^2 &= \langle \alpha_n y_n + (1-\alpha_n)T_n x_n - p, j(x_n - p)\rangle \\ &= \alpha_n \langle y_n - p, j(x_n - p)\rangle + (1-\alpha_n)\langle T_n x_n - p, j(x_n - p)\rangle \\ &\le (1-\alpha_n)\|x_n - p\|^2 + \alpha_n \|y_n - p\|\|x_n - p\|. \end{aligned} \tag{3.1}$$

Thus we obtain from (1.2) and (3.1) that

$$\begin{aligned} \|x_n - p\| &\le \|y_n - p\| \\ &= \|\beta_n(f(x_{n-1}) - p) + (1-\beta_n)(x_{n-1} - p)\| \\ &\le \beta_n \|f(x_{n-1}) - f(p)\| + \beta_n \|f(p) - p\| + (1-\beta_n)\|x_{n-1} - p\| \\ &\le \alpha\beta_n \|x_{n-1} - p\| + \beta_n \|f(p) - p\| + (1-\beta_n)\|x_{n-1} - p\| \\ &= (1-(1-\alpha)\beta_n)\|x_{n-1} - p\| + (1-\alpha)\beta_n \frac{1}{1-\alpha}\|f(p) - p\| \\ &\le \max\left\{ \|x_{n-1} - p\|, \frac{1}{1-\alpha}\|f(p) - p\| \right\}. \end{aligned}$$

By induction

$$\|x_n - p\| \le \max\left\{ \|x_0 - p\|, \frac{1}{1-\alpha}\|f(p) - p\| \right\}, \quad n \ge 0,$$

and $\{x_n\}$ is bounded, so are $\{y_n\}$, $\{T_n x_n\}$ and $\{f(x_n)\}$.

Observe that

$$\begin{aligned} \|x_n - T_n x_n\| &= \alpha_n \|y_n - T_n x_n\| \to 0 (n \to \infty), \\ \|y_n - x_{n-1}\| &= \beta_n \|f(x_{n-1}) - x_{n-1}\| \to 0 (n \to \infty). \end{aligned} \tag{3.2}$$

Set $A_n = (2I - T_n)^{-1}$, it is known that $\{A_n\}_{n=1}^N$ are all nonexpansive mappings and $F(A_n) = F(T_n)$ as a consequence of Theorem 6 of [3]. Then we have

$$\begin{aligned}\|x_n - A_nx_n\| &= \|A_nA_n^{-1}x_n - A_nx_n\| \\ &\leq \|A_n^{-1}x_n - x_n\| \\ &= \|x_n - T_nx_n\|.\end{aligned}$$

It also follows from (3.2) that $\lim\limits_{n\to\infty} \|x_n - A_nx_n\| = 0$.

Next, we claim that

$$\limsup_{n\to\infty}\langle f(p) - p, j(x_n - p)\rangle \leq 0, \quad p \in F, \tag{3.3}$$

where $p = Q(f) = \lim\limits_{t\to 0} z_t$ with z_t being the fixed point of $z \longmapsto tf(z) + (1 - t)A_nz$ (see Lemma 2.1).

Indeed, z_t solves the fixed point equation $z_t = tf(z_t) + (1 - t)A_nz_t$. Then we have $z_t - x_n = (1 - t)(A_nz_t - x_n) + t(f(z_t) - x_n)$. Thus we obtain

$$\begin{aligned}\|z_t - x_n\|^2 &\leq (1 - t)^2\|A_nz_t - x_n\|^2 + 2t\langle f(z_t) - x_n, j(z_t - x_n)\rangle \\ &\leq (1 - t)^2[\|A_nz_t - A_nx_n\| + \|x_n - A_nx_n\|]^2 \\ &\qquad +2t\langle f(z_t) - z_t, j(z_t - x_n)\rangle + 2t\|z_t - x_n\|^2.\end{aligned} \tag{3.4}$$

Noting that

$$\begin{aligned}&\langle f(z_t) - z_t, j(z_t - x_n)\rangle \\ &= \langle f(z_t) - f(p), j(z_t - x_n)\rangle + \langle f(p) - z_t, j(z_t - x_n)\rangle \\ &\leq \alpha\|z_t - p\|\|z_t - x_n\| + \langle f(p) - z_t, j(z_t - x_n)\rangle.\end{aligned}$$

Thus (3.4) gives

$$\begin{aligned}\|z_t - x_n\|^2 &\leq (1 - t)^2[\|z_t - x_n\| + \|x_n - A_nx_n\|]^2 + 2\alpha t\|z_t - p\|\|z_t - x_n\| \\ &\quad + 2t\langle f(p) - z_t, j(z_t - x_n)\rangle + 2t\|z_t - x_n\|^2 \\ &\leq (1 - t)^2\|z_t - x_n\|^2 + a_n(t) + 2\alpha t\|z_t - p\|\|z_t - x_n\| \\ &\quad + 2t\langle f(p) - z_t, j(z_t - x_n)\rangle + 2t\|z_t - x_n\|^2,\end{aligned}$$

where

$$a_n(t) = (2\|z_t - x_n\| + \|x_n - A_nx_n\|)\|x_n - A_nx_n\| \to 0 \ as \ n \to \infty. \tag{3.5}$$

It follows that

$$\langle z_t - f(p), j(z_t - x_n)\rangle \leq \frac{t}{2}\|z_t - x_n\|^2 + \frac{1}{2t}a_n(t) + \alpha\|z_t - p\|\|z_t - x_n\|. \tag{3.6}$$

Letting $n \to \infty$ in (3.6) and noting (3.5) yields

$$\limsup_{n\to\infty}\langle z_t - f(p), j(z_t - x_n)\rangle \leq \frac{t}{2}M + \alpha M\|z_t - p\|. \tag{3.7}$$

where $M > 0$ is a constant.

Since the set $\{z_t - x_n\}$ is bounded, the duality map j is norm-to-norm uniformly continuous on bounded sets of E, and z_t strongly converges to p. By letting $t \to 0$ in (3.7), it is not hard to find that the two limits can be interchanged and (3.3) is thus proven.

Finally, we show that $x_n \to p$ strongly.

Indeed, using Lemma 2.2, we obtain

$$\begin{aligned}
&\|x_n - p\|^2 \leq \|y_n - p\|^2\\
&= \|(1-\beta_n)(x_{n-1} - p) + \beta_n(f(x_{n-1}) - p)\|^2\\
&\leq (1-\beta_n)^2\|x_{n-1} - p\|^2 + 2\beta_n(1-\beta_n)\langle f(x_{n-1}) - p, j(x_{n-1} - p)\rangle\\
&\quad + \max\{(1-\beta_n)\|x_{n-1} - p\|, 1\}\beta_n\|f(x_{n-1}) - p\|b(\beta_n\|f(x_{n-1} - p\|)\\
&\leq (1-\beta_n)^2\|x_{n-1} - p\|^2 + 2\beta_n(1-\beta_n)\langle f(x_{n-1}) - f(p), j(x_{n-1} - p)\rangle\\
&\quad + 2\beta_n(1-\beta_n)\langle f(p) - p, j(x_{n-1} - p)\rangle + M_1\beta_n b(\beta_n)\\
&\leq (1-\beta_n)^2\|x_{n-1} - p\|^2 + 2\beta_n\|f(x_{n-1}) - f(p)\|\|x_{n-1} - p\|\\
&\quad + 2\beta_n(1-\beta_n)\langle f(p) - p, j(x_{n-1} - p)\rangle + M_1\beta_n b(\beta_n)\\
&\leq (1-\beta_n)^2\|x_{n-1} - p\|^2 + 2\alpha\beta_n\|x_{n-1} - p\|\|x_{n-1} - p\|\\
&\quad + 2\beta_n(1-\beta_n)\langle f(p) - p, j(x_{n-1} - p)\rangle + M_1\beta_n b(\beta_n)\\
&\leq [1 - 2(1-\alpha)\beta_n]\|x_{n-1} - p\|^2 + 2\beta_n\langle f(p) - p, j(x_{n-1} - p)\rangle\\
&\quad + M_2\beta_n b(\beta_n) + M_2\beta_n^2,
\end{aligned}$$

where $M_1 > 0$ and $M_2 > 0$ are some constants.

Now we apply Lemma 2.3 and use (3.3) to see that $\|x_n - p\| \to 0$. This completes the proof. $\square$

Remark 3.1. Theorem 3.1 prove the strong convergence in the framework of real uniformly smooth Banach spaces. Our theorem extend Theorem 1.1 to the more general real Banach spaces and also extend Theorem 1.2 without condition $\liminf\limits_{n\to\infty} d(x_n, F) = 0$ and at the same time extend the mapping to pseudocontractive mapping.

Corollary 3.1. *Let E be a uniformly smooth Banach space and let K be a nonempty closed convex subset of E. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of K such that $F = \bigcap\limits_{i=1}^{N} F(T_i) \neq \emptyset$. Let $f \in \Pi_K$, $x_0 \in K$ and $\{\alpha_n\}_{n=1}^\infty, \{\beta_n\}_{n=1}^\infty$ be two sequences in* $(0,1)$ *satisfying the following conditions:*

$$\text{(i) } \alpha_n \to 0, \beta_n \to 0, n \to \infty; \quad \text{(ii) } \sum_{n=0}^{\infty} \beta_n = \infty.$$

Then the sequence $\{x_n\}$ defined by (1.2) *converges strongly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.*

Corollary 3.2. *Let E be a uniformly smooth Banach space and let K be a nonempty closed convex subset of E. Let $\{T_i\}_{i=1}^N$ be N nonexpansive self-maps of K such that $F = \bigcap_{i=1}^{N} F(T_i) \neq \emptyset$. Given a point $u \in K$, $x_0 \in K$ and $\{\alpha_n\}_{n=1}^\infty, \{\beta_n\}_{n=1}^\infty$ be two sequences in $(0,1)$ satisfying the following conditions:*

$$\text{(i) } \alpha_n \to 0, \beta_n \to 0, n \to \infty; \quad \text{(ii) } \sum_{n=0}^{\infty} \beta_n = \infty.$$

Then the sequence $\{x_n\}$ defined by

$$\begin{aligned} y_n &= \beta_n u + (1-\beta_n) x_{n-1}, n \geq 1, \\ x_n &= \alpha_n y_n + (1-\alpha_n) T_n x_n, \end{aligned}$$

converges strongly to a common fixed point of the mappings $\{T_i\}_{i=1}^N$.

Remark 3.2. *Corollary* 3.2 *improve Theorem* 1.3 *without compactness assumption of mapping.*

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