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SOME STABILITY RESULTS FOR TWO HYBRID FIXED POINT ITERATIVE ALGORITHMS OF KIRK-ISHIKAWA AND KIRK-MANN TYPE

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Abstract. In this paper, we prove some stability results for sequences of operators using two newly introduced hybrid fixed point iterative algorithms of Kirk-Ishikawa and Kirk-Mann types in normed linear space by employing a certain class of contractive definition. Our results generalize, extend and improve some of the results of Harder and Hicks [11], Rhoades [29, 30], Osilike [26], Berinde [2, 3] as well as the recent results of the author [12, 23, 24, 25].

1. INTRODUCTION

Let (E, d) be a complete metric space and $T: E \to E$ a selfmap of E. Suppose that $F_T = \{p \in E \mid Tp = p\}$ is the set of fixed points of T. There are several iteration processes in the literature for which the fixed points of operators have been approximated over the years by various authors. In a complete metric space, the Picard iteration process $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = Tx_n, \quad n = 0, 1, \dots \tag{1.1}$$

has been employed to approximate the fixed points of mappings satisfying the inequality relation

$$d(Tx, Ty) \leq \alpha d(x, y), \quad \forall x, y \in E \text{ and } \alpha \in [0, 1). \tag{1.2}$$

Condition (1.2) is called *Banach's contraction condition.* Any operator satisfying (1.2) is called a *strict contraction.* Also, condition (1.2) is significant in the celebrated Banach's fixed point theorem [1].

In the Banach space setting, we shall state some of the iteration processes generalizing (1.1) as follows:
For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$x_{n+1} = (1 - \alpha_n)x_n + \alpha_n Tx_n, \; n = 0, 1, \dots \tag{1.3}$$

where $\{\alpha_n\}_{n=0}^{\infty} \subset [0, 1]$, is called the Mann iteration process (see Mann [21]).

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For $x_0 \in E$, the sequence $\{x_n\}_{n=0}^{\infty}$ defined by

$$\left.\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T z_n \\ z_n &= (1-\beta_n)x_n + \beta_n T x_n \end{aligned}\right\} \quad n = 0, 1, \dots, \tag{1.4}$$

where $\{\alpha_n\}_{n=0}^{\infty}$ and $\{\beta_n\}_{n=0}^{\infty}$ are sequences in $[0,1]$, is called the Ishikawa iteration process (see Ishikawa [14]). See Berinde [3] for detail on various iteration processes.

Kannan [16] established an extension of the Banach's fixed point theorem by using the following contractive definition: For a selfmap T, there exists $\beta \in \left(0, \dfrac{1}{2}\right)$ such that

$$d(Tx, Ty) \leq \beta\left[d(x, Tx) + d(y, Ty)\right], \quad \forall x, y \in E. \tag{1.5}$$

Chatterjea [6] used the following contractive condition: For a selfmap T, there exists $\gamma \in \left(0, \dfrac{1}{2}\right)$ such that

$$d(Tx, Ty) \leq \gamma\left[d(x, Ty) + d(y, Tx)\right], \quad \forall x, y \in E. \tag{1.6}$$

Zamfirescu [38] established a nice generalization of the Banach's fixed point theorem by combining (1.2), (1.5) and (1.6). That is, for a mapping $T\colon E \to E$, there exist real numbers α, β, γ satisfying $0 \leq \alpha < 1$, $0 \leq \beta < \dfrac{1}{2}$, $0 \leq \gamma < \dfrac{1}{2}$ respectively such that for each $x, y \in E$, at least one of the following is true:

$$\left.\begin{aligned} &(z_1)\ d(Tx, Ty) \leq \alpha d(x, y) \\ &(z_2)\ d(Tx, Ty) \leq \beta\left[d(x, Tx) + d(y, Ty)\right] \\ &(z_3)\ d(Tx, Ty) \leq \gamma\left[d(x, Ty) + d(y, Tx)\right]. \end{aligned}\right\} \tag{1.7}$$

A mapping $T\colon E \to E$ satisfying (1.7) is called a *Zamfirescu contraction.* Any mapping satisfying condition (z_2) of (1.7) is called a *Kannan mapping,* while a mapping satisfying condition (z_3) is called a *Chatterjea operator.* The contractive condition (1.7) implies

$$d(Tx, Ty) \leq 2\delta d(x, Tx) + \delta d(x, y), \quad \forall x, y \in E, \tag{1.8}$$

where $\delta = \max\left\{\alpha, \dfrac{\beta}{1-\beta}, \dfrac{\gamma}{1-\gamma}\right\}$, $0 \leq \delta < 1$.

Rhoades [32, 33] used condition (1.7) to obtain some convergence results for Mann and Ishikawa iteration processes in a uniformly convex Banach space, while Berinde [4] extended the results of [32, 33] to an arbitrary Banach space for the same iteration processes.

The following definition of stability of iteration process is due to Harder and Hicks [11].

Definition 1.1 (Harder and Hicks [11])**.** Let (E, d) be a complete metric space, $T\colon E \to E$ a selfmap of E. Suppose that $F_T = \{p \in E \,|\, Tp = p\}$ is the set of fixed points of T. Let $\{x_n\}_{n=0}^{\infty} \subset E$ be the sequence generated by an iteration procedure involving T which is defined by

$$x_{n+1} = f(T, x_n), \quad n = 0, 1, \dots, \tag{1.9}$$

where $x_0 \in E$ is the initial approximation and f is some function. Suppose $\{x_n\}_{n=0}^{\infty}$ converges to a fixed point p of T. Let $\{y_n\}_{n=0}^{\infty} \subset E$ and set

$$\epsilon_n = d(y_{n+1}, f(T, y_n)), \quad n = 0, 1, \ldots.$$

Then, the iteration procedure (1) is said to be T-stable or stable with respect to T if and only if $\lim_{n\to\infty} \epsilon_n = 0$ implies $\lim_{n\to\infty} y_n = p$.

Remark 1.1. Since the metric is induced by the norm, we have

$$\epsilon_n = ||y_{n+1} - f(T, y_n)||, \quad n = 0, 1, \ldots,$$

in place of

$$\epsilon_n = d(y_{n+1}, f(T, y_n)), \quad n = 0, 1, \ldots,$$

in the definition of stability whenever we are working in normed linear space or Banach space.

If in (1.9),

$$f(T, x_n) = Tx_n, \quad n = 0, 1, \ldots,$$

then we have the Picard iteration process defined in (1.1), while we obtain the Ishikawa iteration process (1.4) from (1.9) if

$$f(T, x_n) = (1-\alpha_n)x_n + \alpha_n T z_n,\ z_n = (1-\beta_n)x_n + \beta_n T x_n,\ n = 0, 1, \ldots,\ \alpha_n, \beta_n \in [0, 1].$$

Several stability results established in metric spaces and normed linear spaces are available in the literature. Some of the various authors whose contributions are of colossal value in the study of stability of the fixed point iteration procedures are Ostrowski [28], Harder and Hicks [11], Rhoades [29, 31], Osilike [26], Osilike and Udomene [27], Jachymski [15], Berinde [2, 3] and Singh et al [37]. Harder and Hicks [11], Rhoades [29, 31], Osilike [26] and Singh et al [37] used the method of the summability theory of infinite matrices to prove various stability results for certain contractive definitions. The method has also been adopted to establish various stability results for certain contractive definitions in Olatinwo et al [23, 24]. Osilike and Udomene [27] introduced a shorter method of proof of stability results and this has also been employed by Berinde [2], Imoru and Olatinwo [12], Olatinwo et al [25] and some others. In Harder and Hicks [11], the contractive definition stated in (1.2) was used to prove a stability result for the Kirk's iteration process. The first stability result on $T-$ stable mappings was proved by Ostrowski [28] for the Picard iteration using condition (1.2). In addition to (1.2), the contractive condition in (1.9) was also employed by Harder and Hicks [11] to establish some stabilty results for both Picard and Mann iteration processes. Rhoades [29, 31] extended the stability results of [11] to more general classes of contractive mappings. Rhoades [29] extended the results of [11] to the following independent contractive condition: there exists $c \in [0, 1)$ such that

$$d(Tx, Ty) \leq c \max\{d(x, y), d(x, Ty), d(y, Tx)\}, \quad \forall x, y \in E. \tag{1.10}$$

Rhoades [31] used the following contractive definition: there exists $c \in [0, 1)$ such that

$$d(Tx, Ty) \leq c \max \left\{ d(x, y), \frac{d(x, Tx) + d(y, Ty)}{2}, d(x, Ty), d(y, Tx) \right\}, \quad (1.11)$$

$\forall x, y \in E$.

Moreover, Osilike [26] generalized and extended some of the results of Rhoades [31] by using a more general contractive definition than those of Rhoades [29, 31]. Indeed, he employed the following contractive definition: there exist $a \in [0, 1]$, $L \geq 0$ such that

$$d(Tx, Ty) \leq Ld(x, Tx) + ad(x, y), \quad \forall x, y \in E. \quad (1.12)$$

Osilike and Udomene [27] introduced a shorter method to prove stability results for the various iteration processes using condition (1.12). Berinde [2] established the same stability results for the same iteration processes using the same set of contractive definitions as in Harder and Hicks [11] but the same method of shorter proof as in Osilike and Udomene [27].

More recently, Imoru and Olatinwo [12] established some stability results which are generalizations of some of the results of [2, 11, 26, 27, 29, 31]. In Imoru and Olatinwo [12], the following contractive definition was employed: there exist $a \in [0, 1)$ and a monotone increasing function $\varphi\colon \mathbb{R}_+ \to \mathbb{R}_+$ with $\varphi(0) = 0$, such that

$$d(Tx, Ty) \leq \varphi(d(x, Tx)) + ad(x, y), \quad \forall x, y \in E. \quad (1.13)$$

Condition (1.13) was also employed in Olatinwo et al [23] to establish some stability results in normed linear space setting with an additional condition of continuity imposed on φ. In the next section, we shall state our new iteration processes, some remarks, definition and lemmas which are required in the sequel.

2. PRELIMINARIES

Throughout this paper, we shall assume that F_T is the set of fixed points of T while, $\bigcap_{i=1}^{k} F_{T_i}$ is the set of common fixed points of operators T_i $(i = 1, 2, \ldots, k)$.

In this paper, we shall consider the following definition of stability of iteration process which is an extension of that of Harder and Hicks [11]:

Definition 2.1. Let (E, d) be a complete metric space and $T_i\colon E \to E$ $(i = \overline{1, k})$ selfmaps of E. Suppose that $\bigcap_{i=1}^{k} F_{T_i}$ is the set of common fixed points of operators T_i $(i = \overline{1, k})$. Let $\{x_n\}_{n=0}^{\infty} \subset E$ be the sequence generated by an iteration procedure involving the sequence of operators $\{T_i\}_{i=1}^{k}$ which is defined by

$$x_{n+1} = f(\{T_i\}_{i=1}^{k}, x_n), \quad n = 0, 1, \ldots, \quad (*)$$

where $x_0 \in E$ is the initial approximation and f is some function. Suppose that $\{x_n\}_{n=0}^{\infty}$ converges to a common fixed point p of $\{T_i\}_{i=1}^{k}$ (that is $p \in \bigcap_{i=1}^{k} F_{T_i}$). Let $\{y_n\}_{n=0}^{\infty} \subset E$ and set $\epsilon_n = d(y_{n+1}, f(\{T_i\}_{i=1}^{k}, y_n))$, $n = 0, 1, \ldots$. Then, the iteration procedure $(*)$ is said to be $\{T_i\}_{i=1}^{k}$-stable or stable with respect to the sequence of operators $\{T_i\}_{i=1}^{k}$ if and only if $\lim_{n\to\infty} \epsilon_n = 0$ implies $\lim_{n\to\infty} y_n = p$.

If $i = k = 1$ in the Definition 2.1, then $T_i = T_1 = T$ and this reduces to the definition of stability of iteration process due to Harder and Hicks [11]. Remark 1.1 also holds in this context of stability.

We shall introduce and employ the following iteration processes: Let E be a Banach space, $T_i\colon E \to E$ $(i = \overline{1,k})$ selfmaps of E and $x_0 \in E$. Then, define the sequence$\{x_n\}_{n=0}^{\infty}$ by

$$\left.\begin{aligned} &x_{n+1} = \alpha_{n,0}x_n + \sum_{i=1}^{k} \alpha_{n,i}T_i z_n, \quad \sum_{i=0}^{k} \alpha_{n,i} = 1, \quad n = 0, 1, 2, \ldots, \\ &z_n = \sum_{j=0}^{s} \beta_{n,j}T_j x_n, \quad \sum_{j=0}^{s} \beta_{n,j} = 1, \end{aligned}\right\} \tag{2.1}$$

$k \geq s$, $\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$, $\beta_{n,j} \geq 0$, $\beta_{n,0} \neq 0$, $\alpha_{n,i}, \beta_{n,j} \in [0, 1]$, where k and s are fixed integers and T_0 is an identity operator.

If $s = 0$ in iteration process (2.1), we also obtain the following interesting iteration process in a normed linear space setting:

$$x_{n+1} = \sum_{i=0}^{k} \alpha_{n,\ i}\ T_i x_n, \ \sum_{i=0}^{k} \alpha_{n,\ i} = 1, \quad n = 0, 1, 2, \ldots, \tag{2.2}$$

$\alpha_{n,i} \geq 0$, $\alpha_{n,0} \neq 0$, $\alpha_{n,i} \in [0, 1]$, where k is a fixed integer and T_0 is an identity operator.

The iteration process (2.1) will be called the *general Kirk-Ishikawa* type iteration process while the iteration process (2.2) will be called a *general Kirk-Mann* type iteration process.

(i) If $s = 0$, $k = 1$ in iteration process (2.1), then we have $y_n = \beta_{n,0}x_n = x_n$, $\beta_{n,0} = 1$ and $x_{n+1} = (1 - \alpha_{n,1})x_n + \alpha_{n,1}T_1 x_n$,

which is the usual Mann iteration process with $\sum_{i=0}^{1} \alpha_{n,i} = 1$, $\alpha_{n,1} = \alpha_n$.

(ii) Also, if $s = k = 1$, in iteration process (2.1), we get

$$\left.\begin{aligned} &x_{n+1} = (1 - \alpha_{n,1})x_n + \alpha_{n,1}T_1 y_n \\ &y_n = (1 - \beta_{n,1})x_n + \beta_{n,1}T_1 x_n, \end{aligned}\right\}$$

which is the usual Ishikawa iteration process with $\sum_{i=0}^{1}\alpha_{n,i} = \sum_{i=0}^{1}\beta_{n,i} = 1$, $\alpha_{n,1} = \alpha_n$, $\beta_{n,1} = \beta_n$.

(iii) If $s = 0$, $\alpha_{n,i} = \alpha_i$ and $T_i = T^i$ in iteration process (2.1), then we obtain the usual Kirk iteration process

$$x_{n+1} = \sum_{i=0}^{k}\alpha_i T^i x_n, \quad \sum_{i=0}^{k}\alpha_i = 1, \quad n = 0, 1, 2, \dots, \tag{2.3}$$

with $y_n = \beta_{n,0}x_n = x_n$, since $\beta_{n,0} = 1$.

Eqn. (2.2) is also a generalization of Picard, Schaefer, Mann and the Kirk iteration processes. The iteration processes (2.1) and (2.2) are also generalizations of those in Maiti and Saha [20] and Liu et al [19]. See Berinde [3, 5] for detail on the various existing fixed point iteration processes.

We shall employ the following contractive definition: For selfmaps $T_i\colon E \to E$, $(i = \overline{1,k})$, there exist real numbers $k \geq 0$, $a_i \in [0,1)$, $(i = \overline{1,k})$ and a monotone increasing function $\varphi\colon \mathbb{R}^+ \to \mathbb{R}^+$ with $\varphi(0) = 0$, such that $\forall x, y \in E$,

$$||T_i x - T_i y|| \leq \frac{\varphi(||x - T_i x||) + a_i||x - y||}{1 + k||T_i x - x||}, \tag{2.4}$$

where $T_0 = I$ is an identity operator.

Also, we shall employ the following lemma in the sequel:

Lemma 2.1 (Berinde [2, 3]). *If δ is a real number such that $0 \leq \delta < 1$, and $\{\epsilon_n\}_{n=0}^{\infty}$ is a sequence of positive numbers such that $\lim_{n\to\infty}\epsilon_n = 0$, then for any sequence of positive numbers $\{u_n\}_{n=0}^{\infty}$ satisfying*

$$u_{n+1} \leq \delta u_n + \epsilon_n, \quad n = 0, 1, \dots,$$

we have $\lim_{n\to\infty} u_n = 0$.

It is our purpose in this paper to prove some stability results for the iterative processes defined in (2.1) and (2.2). Our results are improvements, generalizations and extensions of some of the results of Harder and Hicks [11], Rhoades [29, 30], Osilike [26], Osilike and Udomene [27], Berinde [2, 3] as well as the recent results of the author [23, 24, 25].

3. MAIN RESULTS

Theorem 3.1. *Let $(E, ||\cdot||)$ be a normed linear space and $T_i\colon E \to E$ $(i = \overline{1,k})$ selfmaps of E satisfying the contractive condition (2.4), where $T_0 = I$ is an identity operator, $\varphi\colon \mathbb{R}_+ \to \mathbb{R}_+$, a monotone increasing function such that $\varphi(0) = 0$. Let $x_0 \in E$, and $\{x_n\}_{n=0}^{\infty}$ be the general Kirk-Ishikawa iteration process defined by (2.1). Suppose that T_i $(i = \overline{1,k})$ have a common fixed point p. Then, the general Kirk-Ishikawa iteration process is $\{T_i\}_{i=1}^{k}$-stable.*

Proof. Let $\{y_n\}_{n=0}^{\infty} \subset E$, $\epsilon_n = \left\| y_{n+1} - \alpha_{n,0} y_n - \sum_{i=1}^{k} \alpha_{n,\ i} T_i b_n \right\|$ and $b_n = \sum_{r=0}^{s} \beta_{n,r} T_r y_n$. Suppose that $\lim_{n\to\infty} \epsilon_n = 0$. Then, we shall employ both the contractive condition and the triangle inequality to establish that $\lim_{n\to\infty} y_n = p$. Let

$$\left(\sum_{i=1}^{k} \alpha_{n,i} a_i\right)\left(\sum_{r=0}^{s} \beta_{n,r} a_r\right) + \alpha_{n,0} \leq \delta$$

with $0 \leq \delta < 1$ and $a_0 = 1$. Then,

$$\begin{aligned}
||y_{n+1} - p|| &\leq \left\| y_{n+1} - \alpha_{n,0} y_n - \sum_{i=1}^{k} \alpha_{n,i} T_i b_n \right\| + \left\| \alpha_{n,0} y_n + \sum_{i=1}^{k} \alpha_{n,i} T_i b_n - p \right\| \\
&= \left\| \alpha_{n,0} y_n + \sum_{i=1}^{k} \alpha_{n,i} T_i b_n - \sum_{i=0}^{k} \alpha_{n,i} T_i p \right\| + \epsilon_n \\
&= \left\| \sum_{i=1}^{k} \alpha_{n,i} (T_i b_n - T_i p) + \alpha_{n,0}(y_n - p) \right\| + \epsilon_n \\
&\leq \sum_{i=1}^{k} \alpha_{n,i} \|T_i p - T_i b_n\| + \alpha_{n,0}\|y_n - p\| + \epsilon_n \\
&\leq \sum_{i=1}^{k} \alpha_{n,i} \left\{ \frac{\varphi(\|p - T_i p\|) + a_i \|p - b_n\|}{1 + k\|T_i p - p\|} \right\} + \alpha_{n,0}\|y_n - p\| + \epsilon_n \\
&\leq \left[\Big(\sum_{i=1}^{k} \alpha_{n,i} a_i \Big) \Big(\sum_{r=0}^{s} \beta_{n,r} a_r \Big) + \alpha_{n,0} \right] \|y_n - p\| + \epsilon_n \\
&\leq \delta \|y_n - p\| + \epsilon_n. \qquad (3.1)
\end{aligned}$$

Since $0 \leq \delta < 1$, using Lemma 2.1 in (3.1) yields $\lim_{n\to\infty} y_n = p$.

Conversely, let $\lim_{n\to\infty} y_n = p$. Then, we shall show that $\lim_{n\to\infty} \epsilon_n = 0$, using again both the contractive condition and the triangle inequality as follows:

$$\begin{aligned}
\epsilon_n &= \left\| y_{n+1} - \alpha_{n,0} y_n - \sum_{i=1}^{k} \alpha_{n,i} T_i b_n \right\| \leq \|y_{n+1} - p\| + \left\| p - \alpha_{n,0} y_n - \sum_{i=1}^{k} \alpha_{n,i} T_i b_n \right\| \\
&= \|y_{n+1} - p\| + \left\| \sum_{i=0}^{k} \alpha_{n,i} T_i p - \alpha_{n,0} y_n - \sum_{i=1}^{k} \alpha_{n,i} T_i b_n \right\| \\
&= \|y_{n+1} - p\| + \left\| \sum_{i=1}^{k} \alpha_{n,i} (T_i p - T_i b_n) + \alpha_{n,0}(p - y_n) \right\|
\end{aligned}$$

$$\leq \|y_{n+1} - p\| + \sum_{i=1}^{k} \alpha_{n,i}\|T_i p - T_i b_n\| + \alpha_{n,0}\|y_n - p\|$$
$$\leq \|y_{n+1} - p\| + \sum_{i=1}^{k} \alpha_{n,i} \left\{ \frac{\varphi(\|p - T_i p\|) + a_i\|p - b_n\|}{1 + k\|T_i p - p\|} \right\} + \alpha_{n,0}\|y_n - p\|$$
$$\leq \|y_{n+1} - p\| + \left[\Big(\sum_{i=1}^{k} \alpha_{n,i} a_i \Big) \Big(\sum_{r=0}^{s} \beta_{n,r} a_r \Big) + \alpha_{n,0} \right] \|y_n - p\|$$
$$\leq \|y_{n+1} - p\| + \delta\|y_n - p\| \to 0 \text{ as } n \to \infty.$$

□

Theorem 3.2. *Let $(E, ||\cdot||)$ be a normed linear space and $T_i \colon E \to E$ $(i = \overline{1,k})$ selfmaps of E satisfying the contractive condition (2.4), where $T_0 = I$ is an identity operator and $\varphi \colon \mathbb{R}^+ \to \mathbb{R}^+$, a monotone increasing function such that $\varphi(0) = 0$. Let $x_0 \in E$, and $\{x_n\}_{n=0}^{\infty}$ be the general Kirk-Mann iteration process defined by (2.2). Suppose that T_i $(i = \overline{1,k})$ have a common fixed point p. Then, the general Kirk-Mann iteration process is $\{T_i\}_{i=1}^{k}$ −stable.*

Proof. The proof is similar to that of Theorem 3.1. □

Remark 3.1. Theorems 3.1 is a generalization and extension of Theorem 2 of Osilike [26], Theorem 2 and Theorem 5 of Osilike and Udomene [27] as well as Theorem 3 of Olatinwo et al [24]. Theorem 3.2 is a generalization of Theorem 2 of Rhoades [29, 31], Theorem 3 of Berinde [2] as well as Theorem 3.2 of Imoru and Olatinwo [12].

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